Intermediate Algebra (B) **Name** ANSWER

Tri B Free Response Practice Questions ***Class period*** *1 2 3 4 5*

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1. Given the function $f(x) = 3\sqrt{x+3}$

a.) Identify the domain and range of $f(x)$.

D: $x \geq -3$
R: $y \geq 0$

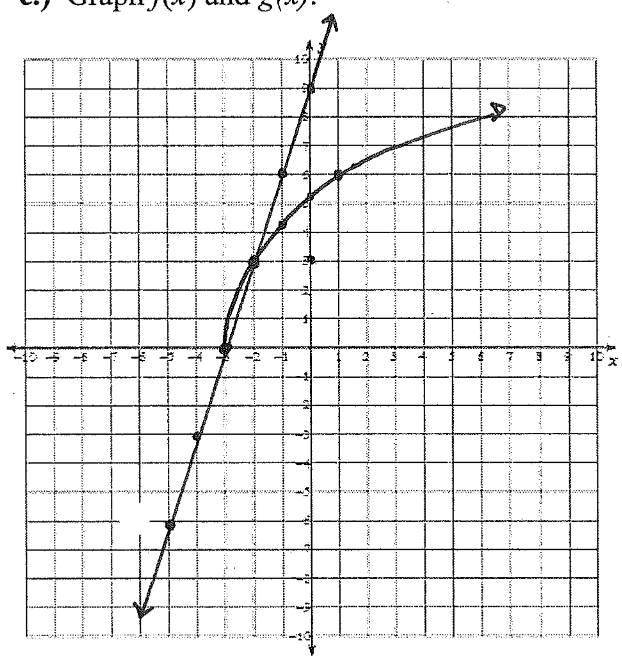
Given the function $g(x) = 3x + 9$

b.) Identify the domain and range of $g(x)$.

D: $\mathbb{R}$
R: $\mathbb{R}$

x	f(x)	g(x)
-5	error	-6
-4	error	-3
-3	0	0
-2	3	3
-1	4.24	6
0	5.20	9
1	6	12

c.) Graph $f(x)$ and $g(x)$.



WORK SPACE

$$\frac{3\sqrt{x+3}}{3} = \frac{3x+9}{3}$$

$$\left(\sqrt{x+3}\right)^2 = (x+3)^2$$

$$x + 3 = x^2 + 6x + 9$$
$$-x \qquad -x$$
$$3 = x^2 + 5x + 9$$
$$-3 \qquad -3$$
$$0 = x^2 + 5x + 6$$
$$0 = (x+3)(x+2)$$

$x + 3 = 0$, -3 -3, $x = -3$ $\qquad$ $x + 2 = 0$, -2 -2, $x = -2$

c.) Solve the equation $f(x) = g(x)$ algebraically.

$x = -3, -2$

Check: look at the graph!

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2. AirBuilder manufactures airplanes. The company does not produce many planes but they are very expensive. Many things effect the cost of doing business including but not limited to building rental, utilities, equipment purchases, supplies needed for construction, employee salaries, and health and dental benefits. The cost of producing x planes is given by the polynomial function $c(x) = 0.5x^4 - 14.5x^3 + 128x^2 + 288$ where x represents the number planes that are produced and $c(x)$ represents the cost (in millions of dollars) to make x planes. The company sells the airplanes for 402 million dollars. By government contract, AirBuilder cannot build more than 17 planes per year.

WORK SPACE

a.) Let $r(x)$ represent the revenue (total income) the company gets from selling x planes. Write the function for $r(x)$.

If it sells for 402 mil $ per plane that would be 402 times the # of planes sold

$r(x) = 402x$

b.) Let $p(x)$ represent the profit the company makes from selling x planes. Write a polynomial function for $p(x)$. Profit = revenue – cost so $p(x) = r(x) - c(x)$.

$(402x) - (0.5x^4 - 14.5x^3 + 128x^2 + 288)$

$-0.5x^4 + 14.5x^3 - 128x^2 + 402x - 288$

$p(x) = -0.5x^4 + 14.5x^3 - 128x^2 + 402x - 288$

c.) Find the y-intercept of the profit function. Explain the meaning of this point in the context of the problem.

The y-intercept is where x=0 on the graph (0,-288)

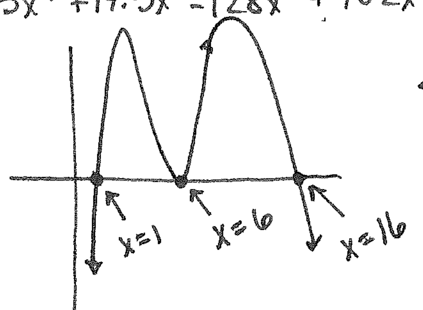
(0,-288) or y=-288
This means if 0 planes are sold the manufacturers lose $288 million.

d.) Find the x-intercept(s) of the profit function. Explain the meaning of this point in the context of the problem.

The x-intercepts are where y=0

$0 = -0.5x^4 + 14.5x^3 - 128x^2 + 402x - 288$

Graph:



(1,0) and (6,0) (16,0) or x=1 and x=6 and x=16
This means if they only manufacture 1 plane, 6 planes or 16 planes the profit is $0.

e.) Harriet Jones is the CEO of AirBuilder. She wants to maximize the profit of the company. How many planes should she plan to construct? What is the maximum possible profit of the company? (You cannot build a fractional component of an airplane.)

x	y
0	-288
1	0
2	112
3	117
4	72
5	22
6	0
7	27
8	112
9	252
10	432
11	625
12	792
13	882
14	832
15	567
16	0

← max profit (13, 882)

* since we can't use decimals just look through the table of points

13 planes
$882 million profit

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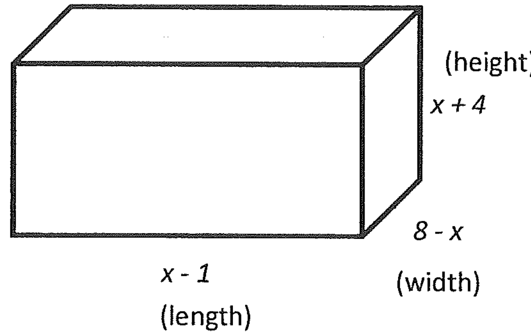
3. The volume (in^3) of a tissue box can be given by the function $V(x) = (8 - x)(x + 4)(x - 1)$.

a) Explain why $1 < x < 8$. (x must be greater than 1 and less than 8.) Be very detailed.

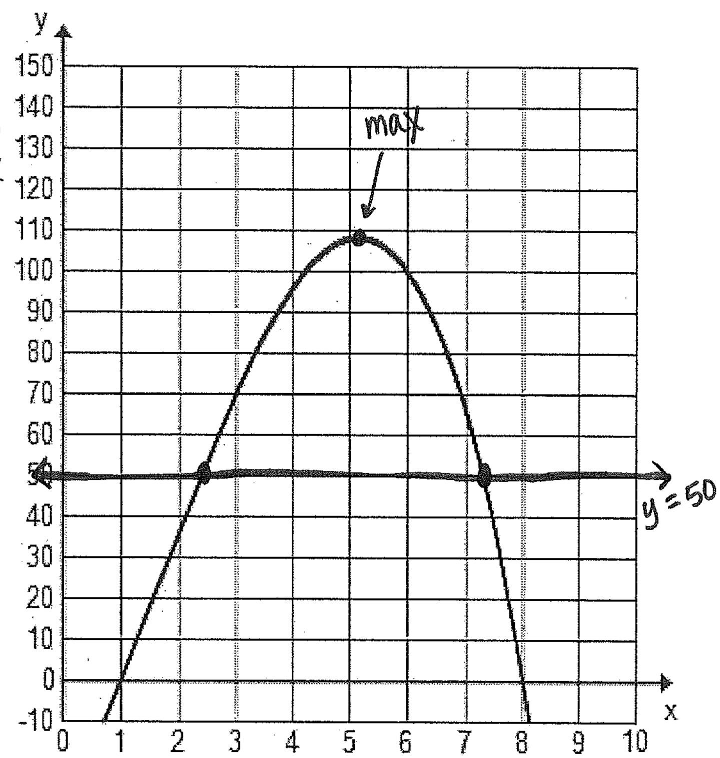
If x is less than 1 or greater than 8 then you would end up with negative side lengths and volume. If x = 1 or x = 8 then one side would be 0 and make the volume 0.



b) What value of x produces the largest volume of the tissue box? Round to the nearest tenth.

max — graph V(x) and trace the max

X = 5.1 inches



c) What is the maximum volume of the tissue box? Round to the nearest tenth.

$V = (8-5.1)(5.1+4)(5.1-1) = 108.2$

$V = 108.2\ in^3$

d) What are the **dimensions** of this box with the maximum volume? Round to the nearest tenth.

length = 5.1 - 1 = 4.1 in

width = 8 - 5.1 = 2.9 in

height = 5.1 + 4 = 9.1 in

e) Write the equation in standard form.

$(8-x)(x+4)(x-1)$

$8x + 32 - x^2 - 4x$

$(-x^2 + 4x + 32)(x-1)$

$(-x^2 + 4x + 32)(x - 1)$

$-x^3 + 1x^2 + 4x^2 - 4x + 32x - 32$

$-x^3 + 5x^2 + 28x - 32$

$V(x) = -x^3 + 5x^2 + 28x - 32$

f) What value(s) of x produce a volume of 50 in^3? Mark the graph to indicate your findings. (Round to the nearest tenth.)

→ V = 50

$50 = (8-x)(x+4)(x-1)$

Graph y = 50 and y = (8-x)(x+4)(x-1) and see where the lines intersect.

X = 2.4 in

X = 7.3 in

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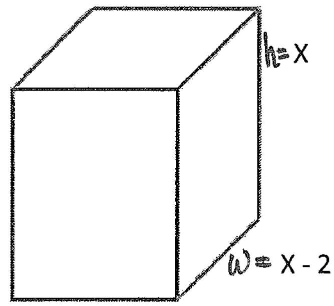
4. You are building a storage box for your sports equipment. The width of the box is 2 feet less than the height of the box and the length is 5 feet more than the height of the box.

$V = l \cdot w \cdot h$



$l = x + 5$

$V = (x+5)(x-2)(x)$

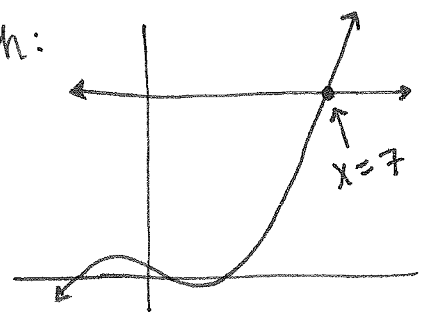
$x^2 - 2x + 5x - 10$

$(x^2 + 3x - 10)(x)$

$V = x^3 + 3x^2 - 10x$

$420 = x^3 + 3x^2 - 10x$

Graph:



length = 7 + 5 = 12 ft
width = x − 2 = 5 ft
height = 7 ft

a) Write a polynomial equation representing the volume of the box in standard form.

$V = x^3 + 3x^2 - 10x$

b) If you need 420 cubic feet of storage, what are the dimensions of the box?

12 ft by 5 ft by 7 ft

c) In part a) you should have gotten a cubic equation, so why in part b) was there only one solution that you could use. Why could you not use the other solutions?

The other two solutions are complex (imaginary).

5. A quadratic equation is given by $5x^2 + bx + 5 = 0$

a) Choose a value for b such that the equation has two real solutions. Explain your reasoning.

WORK SPACE

2 real solutions means the disc is positive...

$b^2 - 4(5)(5) \rightarrow b^2 - 100$

so b^2 has to be more than 100

so b has to be more than 10 or less than -10

$b = 11$

The discriminant would be a positive 21.

b) Choose a value for b such that the equation has two imaginary solutions. Explain your reasoning.

2 imaginary solutions means the disc is negative...

so b^2 has to be less than 100

so b has to be less than 10 or more than -10

$b = 6$

The discriminant would be a negative 64.

c) Choose a value of b such that the equation has one real solution. Explain your reasoning.

One real solution means the disc has to equal 0...

So b^2 has to equal 100

so b has to equal 10

$b = 10$

The discriminant would be exactly 0.

d) Choose a value for b such that the equation could be solved by factoring.

For the equation to be factorable the discriminant needs to be a positive perfect square or 0.

$b = 10$

The discriminant is 0 and

$5x^2 + 10x + 5 = (5x + 5)(x + 1)$

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6. The height of a tossed ball with respect to time can be modeled by the quadratic function $h(t) = -16t^2 + v_0 t + h_0$ where: v_o = initial velocity h_o = initial height
A quarterback throws a football at an initial height of 12 feet with an initial velocity of 80 feet per second.

WORK SPACE

$-16(4)^2 + 80(4) + 12 = 76$

$108 = -16t^2 + 80t + 12$
$-108 \qquad -108$
$0 = -16t^2 + 80t - 96$

$a = -16$, $b = 80$, $c = -96$

$t = \frac{-(80) \pm \sqrt{256}}{2(-16)}$

$t = \frac{-80 \pm 16}{-32}$ → $\frac{-64}{-32} = 2$ → $\frac{-96}{-32} = 3$

x	y
0	
1	76 ← $-16(1)^2 + 80(1) + 12$
2	108 ← part c.
3	108 ← part c.
4	76 ← part b.
5	

*the max should be more than 108 and because this should be symmetric the max is at x=2.5

$-16(2.5)^2 + 80(2.5) + 12 = 112$

$0 = -16t^2 + 80t + 12$

$a = -16$, $b = 80$, $c = 12$

$t = \frac{-(80) \pm \sqrt{7168}}{2(-16)}$

$t = \frac{-80 \pm 84.66}{-32}$ → $\frac{4.66}{-32} = -0.15$ → $\frac{-164.66}{-32} = 5.15$

The -0.15 makes no sense because time isn't negative

a.) Write an equation that models the height of the football with respect to time.

$h(t) = -16t^2 + 80t + 12$

b.) How high is the football after 4 seconds?

76 feet

c.) When will the football be 108 feet high?

2 seconds and 3 seconds

d.) When will the football reach its maximum height?

What is the maximum height of the football?

2.5 seconds

112 feet

e.) When will the football hit the ground if no one catches it? → height of 0

5.15 seconds